ATMOSPHERIC AND OCEANIC SCIENCE LETTERS, 2011, VOL. 4, NO. 3, 131–135

On the Significance and Use of the Generalized Moist Potential Vorticity Equation

FU Shen-Ming[1] and SUN Jian-Hua[2]

[1] *International Center for Climate and Environment Sciences, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China*

[2] *Laboratory of Cloud-Precipitation Physics and Severe Storms, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China*

Received 22 December 2010; revised 25 February 2011; accepted 25 February 2011; published 16 May 2011

Abstract In this paper, the continuity and thermodynamic equations including moisture forcings were derived. Using these two equations and the basic momentum equation of local Cartesian coordinates, the budget equation of generalized moist potential vorticity (GMPV) was derived. The GMPV equation is a good generalization of the Ertel potential vorticity (PV) and moist potential vorticity (MPV) equations. The GMPV equation is conserved under adiabatic, frictionless, barotropic, or saturated atmospheric conditions, and it is closely associated with the horizontal frontogenesis and stability of the real atmosphere. A real case study indicates that term diabatic heating could be a useful diagnostic tool for heavy rainfall events.

Keywords: potential vorticity, moist potential vorticity

Citation: Fu, S.-M., and J.-H. Sun, 2011: On the significance and use of the generalized moist potential vorticity equation, *Atmos. Oceanic Sci. Lett.*, **4**, 131–135.

1 Introduction

Vorticity, the microscopic measure of rotation in fluids, is a dynamical factor that can be defined as the curl of velocity. In the real atmosphere, the dynamical and thermal processes are always combined together. As a result, it is necessary to consider the dynamical and thermal processes together. Rossby (1940) showed a simple form of the potential vorticity (PV) for barotropic motion, and used it to analyse atmospheric patterns. Ertel (1942) proposed a more generalized form of PV, which is strictly conserved for the frictionless, adiabatic motion. The Ertel PV has since been used in many aspects (Robinson, 1989; Thorpe, 1990; Davis, 1992; Montgomery and Farrell, 1992; Martin and Marsili, 2002) for different purposes. Ertel PV is actually a kind of “dry” PV, but in the real atmosphere, moisture is very important, and including moisture in the PV equation is therefore necessary. Many scholars have derived moist potential vorticity (MPV) equations (Bennetts and Hoskins, 1979; Zhang and Cho, 1992; Wu et al., 1995; Gao et al., 2002; Cui et al., 2003), studied the significance of the equations, and used the MPV to diagnose real cases (e.g., diagnosing vortices, heavy rainfall events, etc.). However, among the equations cited above, moisture forcings are only included in the continuity equation, and the form of continuity equation is complex. The purpose of this paper is to improve the MPV equation by considering moisture forcings in both the air continuity and the thermodynamic equations, study the significance of all terms, and diagnose real cases using the equation.

Corresponding author: FU Shen-Ming, fusm@mail.iap.ac.cn

2 Continuity equation including moisture influences

The mass conservation of dry air in the volume element can be described as

$$\frac{\mathrm{d}[\delta m(1-q)]}{\mathrm{d}t}=0 \quad , \tag{1}$$

where the operator $\frac{\mathrm{d}}{\mathrm{d}t}=(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla)$, $\boldsymbol{V}=u\boldsymbol{i}+v\boldsymbol{j}+w\boldsymbol{k}$ is the velocity vector, $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ represents the unit vector points to the east, north, and zenith, respectively, the op- erator $\nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}+\frac{\partial}{\partial z}\boldsymbol{k}$, $\delta m=\rho\delta V$ is the mass within the volume element (ρ is the mass density and $\delta V=\delta x\cdot\delta y\cdot\delta z$ is an infinitesimal volume element of the real atmosphere) and q is the specific humidity. Eq. (1) could be rewritten as $\frac{1}{\delta m}\frac{\mathrm{d}\delta m}{\mathrm{d}t}=\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}+\frac{1}{\delta v}\frac{\mathrm{d}\delta v}{\mathrm{d}t}$. Since $\frac{1}{\delta v}\frac{\mathrm{d}\delta v}{\mathrm{d}t}=\nabla\cdot\boldsymbol{V}$, we can obtain the moist air continuity equation in the following form:

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}+\nabla\cdot\boldsymbol{V}=\frac{1}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}, \tag{2}$$

where the term on the right hand side (RHS), $\frac{1}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}$, represents the effects of moisture variation. If the moisture remains conserved or there is no moisture in the process, Eq. (2) can be rewritten as $\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}+\nabla\cdot\boldsymbol{V}=0$, which is the normal continuity equation. From Eq. (2), it is obvious that, moisture variation works as mass sources or sinks and it influences not only the mass density but also the divergence. As a result, the influence of moisture variation is important, and considering moisture forcings in the real atmosphere is necessary.

3 The thermodynamic equation including moisture influences

The first law of thermodynamics can be described as follows: for a thermodynamic system in equilibrium, the variation of internal energy is equal to the heat absorbed by the system minus the work done by the system. In this paper, the sum of the internal energy and the macroscopic kinetic energy is defined as the prompt energy (PE), namely $K=(I+\frac{1}{2}\boldsymbol{V}\cdot\boldsymbol{V})$, where I stands for the internal energy. Using the above definition, the first law of thermodynamics can also be described as: within a volume element, the variation of the PE is equal to the summation of the heat absorbed and the work received, as shown in Eq. (3) below:

$$\frac{\mathrm{d}[(I+\boldsymbol{V}\cdot\boldsymbol{V}/2)\rho\delta v]}{\mathrm{d}t}=-p\delta v\nabla\cdot\boldsymbol{V}-\delta v\boldsymbol{V}\cdot\nabla p+(\boldsymbol{V}\cdot\boldsymbol{g}+\boldsymbol{V}\cdot\boldsymbol{F})\rho\delta v+\varepsilon\rho\delta v+\dot{Q}\rho\delta v-\lambda L\frac{\mathrm{d}(\rho\delta vq)}{\mathrm{d}t}, \quad (3)$$

where the internal energy per unit mass $I=C_{\mathrm{v}}T$, C_{v} is the constant-volume specific heat of the real atmosphere ($C_{\mathrm{v}}=(1+0.93q)C_{\mathrm{vd}}$, $C_{\mathrm{vd}}=718$ J kg^{-1} K^{-1} is the constant-volume specific heat of dry air), T is temperature (units: K), p is atmospheric pressure, $\boldsymbol{g}$ is the acceleration vector of gravity, $\boldsymbol{F}=\frac{1}{\rho}(\frac{\partial\boldsymbol{\tau}_x}{\partial x}+\frac{\partial\boldsymbol{\tau}_y}{\partial y}+\frac{\partial\boldsymbol{\tau}_z}{\partial z})$ is the molecular viscous force per unit mass, $\varepsilon=\frac{1}{\rho}(\boldsymbol{\tau}_x\cdot\frac{\partial\boldsymbol{V}}{\partial x}+\boldsymbol{\tau}_y\cdot\frac{\partial\boldsymbol{V}}{\partial y}+\boldsymbol{\tau}_z\cdot\frac{\partial\boldsymbol{V}}{\partial z})$, $\dot{Q}$ is the heat rate from extraneous sources, λ is absorptivity of condensation heat (with $0\leqslant\lambda\leqslant1$), and $L=(3164.3–2.4T)$ J g^{-1} is the latent heat of condensation per unit mass. The last term of the RHS, namely $-\lambda L\frac{\mathrm{d}(\rho\delta vq)}{\mathrm{d}t}$, is the latent heat of the phase change from gas to liquid within the volume element and requires that $\mathrm{d}q/\mathrm{d}t\leqslant0$. The simplified momentum equation in Cartesian coordinates (Holton, 1992) is shown in Eq. (4), below:

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}=-\frac{1}{\rho}\nabla p-f\boldsymbol{k}\times\boldsymbol{V}+\boldsymbol{g}+\boldsymbol{F}, \quad (4)$$

where $f=2\Omega\sin\varphi$ is the Coriolis force parameter, Ω is the palstance of the Earth's rotation, and φ is latitude. By taking $\boldsymbol{V}$·Eq. (4) and subtracting it from Eq. (3), we can obtain Eq. (5):

$$\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{K}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}=-\frac{p}{\rho}\nabla\cdot\boldsymbol{V}+\varepsilon+\dot{Q}-\frac{\lambda L}{\rho\delta v}\frac{\mathrm{d}(\rho\delta vq)}{\mathrm{d}t}. \quad (5)$$

The continuity equation Eq. (2) is then taken into Eq. (5) to form Eq. (6), where α is the specific volume:

$$\frac{\mathrm{d}I}{\mathrm{d}t}+p\frac{\mathrm{d}\alpha}{\mathrm{d}t}=\varepsilon+\dot{Q}-\frac{\lambda L}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}-\frac{K}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}-p\alpha\frac{1}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}. \quad (6)$$

It should be noted that the third term of RHS of Eq. (6), $\frac{-\lambda L}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}$, is the heat absorbed from the phase change from gas to liquid within the air volume element ($\mathrm{d}q/\mathrm{d}t\leqslant0$ required). The fourth term, ($-\frac{K}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}$), represents the loss of PE caused by the phase change, and the last term on the right, ($-p\alpha\frac{1}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}$), is the loss of work caused by phase change. In order to simplify Eq. (6), a concise scale analysis was implemented by setting λ=1, indicating an assumption that all heat released during the phase change within the volume element was absorbed. The magnitudes of the terms are then as follows (parts common to all terms are not included):

$\frac{-L}{1-q}\sim10^6$ J kg^{-1}; $\frac{K}{1-q}\sim10^5$ J kg^{-1}; $p\alpha\sim10^5$ J kg^{-1}.

As a result, we can only retain the term $\frac{-L}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}$, which is abbreviated to PC for the remainder of the analysis. The thermodynamic equation including moisture forcings can be then be written as shown in Eq. (7):

$$\frac{\mathrm{d}(C_pT)}{\mathrm{d}t}-\alpha\frac{\mathrm{d}p}{\mathrm{d}t}=\varepsilon+\dot{Q}+PC, \quad (7)$$

where $C_p=C_{\mathrm{v}}+R=(1+0.84q)C_{p\mathrm{d}}$ is the specific heat of the real atmosphere at constant pressure, and $C_{p\mathrm{d}}$ = 1005 J kg^{-1} K^{-1} is the specific heat of dry air at constant pressure. Supposing an adiabatic, frictionless process, Eq. (7) can be used to show that $\frac{\mathrm{d}(C_pT)}{\mathrm{d}t}-\alpha\frac{\mathrm{d}p}{\mathrm{d}t}=0$, which can then be rewritten as $\frac{\mathrm{d}\ln T}{\mathrm{d}t}=\frac{R}{C_p}\frac{\mathrm{d}\ln p}{\mathrm{d}t}$, where $R=R_{\mathrm{d}}(1+0.6\mathrm{l}q)$ is the universal gas constant of the real atmosphere and R_{d}=287 J kg^{-1}K^{-1} is the universal gas constant of dry air. For the real atmosphere, $0.283\leqslant R/C_p\leqslant0.286$, thus R/C_p can be taken as approximately constant. From this assumption, we derive $\int_\theta^T\mathrm{d}\ln T=\frac{R}{C_p}\int_{p_0}^{p}\mathrm{d}\ln p$, where p_0 is the normal atmospheric pressure (generally p_0 = 1000 hPa) and θ is the potential temperature. From the derivation above, the potential temperature of the real atmosphere in Eq. (7) is conserved for adiabatic, frictionless motion. In this paper, friction was not taken into consideration. We then defined the equivalent potential temperature as $\theta_{\mathrm{e}}=\theta\exp(\frac{Lq}{C_pT})$. Eq. (7) can then be rewritten as Eq. (8), shown below:

$$\frac{\mathrm{d}\theta_{\mathrm{e}}}{\mathrm{d}t}=\frac{\theta_{\mathrm{e}}}{C_pT}(\dot{Q}-\frac{qL}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t}). \quad (8)$$

From Eq. (8), it is obvious that for a frictionless, adiabatic process, the equivalent potential temperature θ_{e} is conserved, because the terms on the RHS of Eq. (8) are equal to zero. If there is no moisture phase change within the air volume element, Eq. (8) can be rewritten as $\frac{\mathrm{d}\theta_e}{\mathrm{d}t}=\frac{\theta_e}{C_pT}\dot{Q}$. This form is the same as the thermodynamic equation derived by Wu et al. (1995). As Eq. (8) shows, the terms in the RHS include two parts: (a) the heat from the phase change within the volume element and (b) the heat from extraneous sources (sensible heating, radiation and so on). With this classification, Eq. (8) could be used in a more efficient way: in synoptic processes with heavy rainfall, heating from the phase change could be the major term (especially important for meso-scale systems), whereas, in processes without heavy rainfall, the heat from extraneous sources may be the major term (especially important for large scale systems).

4 The generalized moist potential vorticity equation

Relative vorticity is defined as
$\boldsymbol{\zeta}=\nabla\times\boldsymbol{V}=(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z})\boldsymbol{i}+(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x})\boldsymbol{j}+(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y})\boldsymbol{k}$.

Using $\nabla\times$Eq. (4), the vorticity equation can be written as:

$$\frac{\partial\boldsymbol{\zeta}}{\partial t}+\nabla\times(\boldsymbol{V}\cdot\nabla\boldsymbol{V}+f\boldsymbol{k}\times\boldsymbol{V})=-\nabla\alpha\times\nabla p+\nabla\times\boldsymbol{F}, \tag{9}$$

where $\boldsymbol{\zeta}_a=(\boldsymbol{\zeta}+f\boldsymbol{k})$ (the absolute vorticity vector), $P_e=\frac{\alpha\boldsymbol{\zeta}_a\cdot\nabla\theta_e}{1-q}$ (generalized moist potential vorticity (GMPV)) and $\tilde{Q}=\frac{\theta_e}{C_pT}(\dot{Q}-\frac{qL}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t})$ (diabatic heating). Using Eq. (8) and Eq. (9) and ignoring friction, the budget equation of GMPV can be finally obtained, as shown in Eq. (10). From Eq. (10), we see that variation of GMPV is closely related to diabatic heating and the solenoid:

$$\frac{\mathrm{d}P_e}{\mathrm{d}t}=\underbrace{\frac{\alpha}{1-q}\boldsymbol{\zeta}_a\cdot\nabla\tilde{Q}}_{\text{Diabatic heating}}+\underbrace{\frac{\alpha}{(1-q)}\nabla\theta_e\cdot(\nabla p\times\nabla\alpha)}_{\text{Solenoid}}. \tag{10}$$

5 Significance of the budget equation of the MPV

The absolute vorticity $\boldsymbol{\zeta}_a$ and the gradient of equivalent potential temperature $\nabla\theta_e$ codetermine the sign of the GMPV. The absolute vorticity vector can be deconstructed into horizontal and vertical components, as $\boldsymbol{\zeta}_a=\boldsymbol{\zeta}_h+(\boldsymbol{\zeta}_v+f\boldsymbol{k})$, where $\boldsymbol{\zeta}_h$ is the horizontal component and $\boldsymbol{\zeta}_v$ is the vertical component. Similarly, the gradient of equivalent potential temperature can be deconstructed to $\nabla\theta_e=\nabla_h\theta_e+\frac{\partial\theta_e}{\partial z}\boldsymbol{k}$, where $\nabla_h\theta_e$ is closely related to the horizontal frontogenesis, and $\frac{\partial\theta_e}{\partial z}$ is the conditional stability of the real atmosphere. (a) If there is no obvious frontogenesis (namely $|\nabla_h\theta_e|<<|\frac{\partial\theta_e}{\partial z}|$), the term $\nabla_h\theta_e$ can be neglected, such that $P_e=\frac{\alpha(\zeta_v+f)\partial\theta_e/\partial z}{1-q}$. If the real atmosphere is inertia stable, such that $\zeta_v+f>0$ (we only consider the north hemisphere), the GMPV would be a sign of the stability of the real atmosphere. (b) If the real atmosphere is approximately neutral ($\frac{\partial\theta_e}{\partial z}\approx 0$), as is always the case during heavy rainfall (Fu et al., 2010), then $P_e=\frac{\alpha\boldsymbol{\zeta}_h\cdot\nabla_h\theta_e}{1-q}$. It is obvious that, under a condition of neutral layers, the GMPV is closely related to the front zone while independent of the intensity of the vortex, reflected by ζ_v.

If there is no moisture phase change within the volume element ($\mathrm{d}q/\mathrm{d}t=0$), Eq. (10) can be simplified to the MPV equation derived by Wu et al. (1995). If there is no moisture (dry air, where q=0), Eq. (10) can be simplified to the PV equation derived by Ertel (1942). As a result, Eq. (10) is a good generalization of both PV and MPV budget equations.

For adiabatic ($\tilde{Q}=0$), frictionless processes, Eq. (10) can be simplified to $\frac{\mathrm{d}P_e}{\mathrm{d}t}=\frac{\alpha}{(1-q)}\nabla\theta_e\cdot(\nabla p\times\nabla\alpha)$, where $(\nabla p\times\nabla\alpha)$ is the solenoid. If $(\nabla p\times\nabla\alpha)=0$ or the vector $(\nabla p\times\nabla\alpha)\perp\nabla\theta_e$, the GMPV is conserved. (a) Under the condition of $(\nabla p\times\nabla\alpha)=0$, $\nabla p//\nabla\alpha$, indicating that the real atmosphere is barotropic. (b) In a baroclinic atmosphere, where $(\nabla p\times\nabla\alpha)\neq 0$, $(\nabla p\times\nabla\alpha)\perp\nabla\theta_e$ indicates that the vector $\nabla p\times\nabla\alpha$ is parallel to or within the isentropic surface (Fig. 1). That is, the intersection of the isobaric and isasteric surfaces is parallel with or contained within the isentropic surface (Fig. 1), and this is the saturated situation of the real atmosphere (Wu et al., 1995). From the analysis above, the GMPV is conserved in frictionless, adiabatic, barotropic or saturated processes.

6 Dignostic analysis using the GMPV budget equation

The moist potential vorticity equation is a useful diagnostic tool for heavy rainfall (Gao et al., 2002). In this section, a heavy rainfall event (0000 UTC 4 September

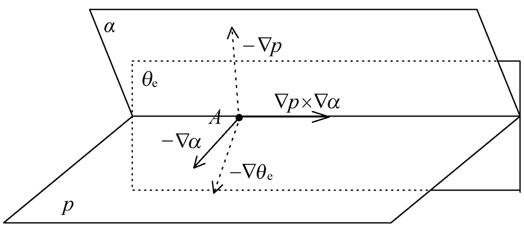

Figure 1 Configuration of the isobaric (p), isasteric (α), and isentropic surfaces (θ_e), where “A” indicates an arbitrary point in the real atmosphere, and the arrows indicate vectors.

2004–0000 UTC 6 September 2004) caused by a southwest vortex (Zhao and Fu, 2007) was analyzed using the GMPV budget equation and reanalysis data from National Centers for Environmental Prediction (NCEP), with a spatial resolution of 1°×1°. The adiabatic heating term in Eq. (10), $\frac{\alpha}{(1-q)}\zeta_{\mathrm{a}}\cdot\nabla\tilde{Q}$, was used to analyze the heavy rainfall. During the calculation, only the heat from the phase change, $\tilde{Q}=\frac{\theta_{\mathrm{e}}}{C_p T}(-\frac{qL}{1-q}\frac{\mathrm{d}q}{\mathrm{d}t})$, was considered. The results of this calculation are shown in Fig. 2. From Figs. 2a and 2c, we see that the rainfall centers in the north as well as southwest of Chongqing and northwest of Guizhou approximately match the centers of the adiabatic heating term; however, the rain centers in the southeast and south of Sichuan differ from the centers of the adiabatic heating term slightly (the southward movement of the rainfall region may be caused by a north wind). As Figs. 2b and 2d depict, the rainfall centers in the south of Sichuan, the northwest and southeast of Chongqing, as well as the north and northwest of Guizhou match the centers of the adiabatic heating term well. From the analysis above, it can be seen that the diabatic heating term is a useful tool to diagnose the regions, centers, and trends of meso-scale heavy rainfall.

7 Conclusions

Taking the view that dry air is conserved in a volume element, a continuity equation containing moisture forcings was derived. From the first law of thermodynamics, a thermodynamic equation including moisture forcings was obtained, and the equation was simplified through a concise scale analysis. With this thermodynamic equation, the potential temperature and equivalent potential temperature of the real atmosphere is conserved in adiabatic, fric-

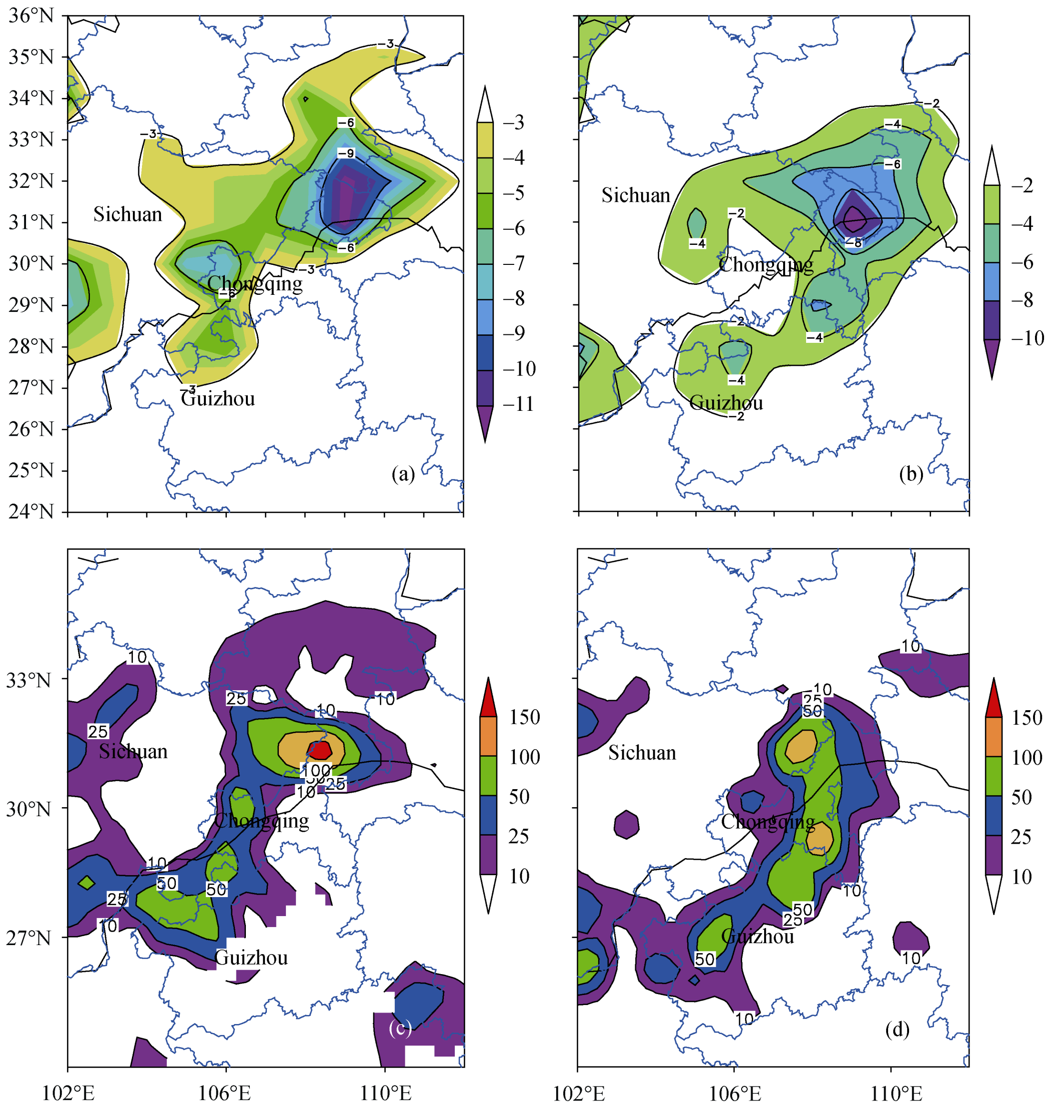

Figure 2 The diabatic heating term (units: 10^{-8} PVU s^{-1}, 1 PVU=10^{-6} m^2 K kg^{-1} s^{-1}) and observed 24-hour precipitation (units: mm). Panels (a) and (b) are the diabatic heating term at 500 hPa averaged during 0000 UTC 4 September–0000 UTC 5 September and 0000 UTC 5 September–0000 UTC 6 September respectively. Panels (c) and (d) are precipitation during 0000 UTC 4 September–0000 UTC 5 September and 0000 UTC 5 September–0000 UTC 6 September respectively.

tionless motions.

Using the continuity equation, the thermodynamic equation including moisture forcings, and the basic momentum equation of the local Cartesian coordinates, the budget equation of GMPV was derived. The budget equation of GMPV indicates that the variation of GMPV is primarily related to the diabatic heating and to the solenoid, which in turn is closely related to baroclinity. Under dry air conditions, the budget equation of GMPV can be simplified to the Ertel potential vorticity equation, and under a condition of no phase changes, the budget equation can be simplified to the MPV equation derived by Wu et al. (1995).

The GMPV is conserved in adiabatic, frictionless barotropic, or saturated atmospheres, and it is closely related to horizontal frontogenesis and the stability of atmospheric layers. A real case study indicates that term diabatic heating could be a good diagnostic tool for heavy rainfall events. In future studies, more cases will be analyzed with the GMPV equation to verify its ability to diagnose characteristics of severe weather events.

Acknowledgements. This research was supported by the National Natural Science Foundation of China (Grant No. 41075032), Chinese Special Scientific Research Project for Public Interest (Grant No. GYHY200906004), and the National Basic Research Program of China (Grant No. 2010CB951804).

References

Bennetts, D. A., and B. J. Hoskins, 1979: Conditional symmetric instability—A possible explanation for frontal rain band, *Quart. J. Roy. Meteor. Soc.*, **105**, 945–962.

Cui, X.-P., S.-T. Gao, and G.-X. Wu, 2003: Moist potential vorticity and up-sliding slantwise vorticity development, *Chinese Phys. Lett.*, **20**, 167–169.

Davis, C. A., 1992: Piecewise potential vorticity inversion, *J. Atmos. Sci.*, **49**, 1397–1411.

Ertel, H., 1942: Ein Neuer hydrodynamischer Wirbelsätze, *Meteor. Z.*, **59**, 271–281.

Fu, S.-M., S.-X. Zhao, and J.-H. Sun, 2010: One kind of vortex causing heavy rainfall during pre-rainy season in South China, *Chinese J. Atmos. Sci.* (in Chinese), **34**(2), 235–252.

Gao, S.-T., T. Lei, and Y.-S. Zhou, 2002: Moist potential vorticity anomaly with heat and mass forcings in torrential rain systems, *Chinese Phys. Lett.*, **19**, 878–880.

Holton, J. R., 1992: *An Introduction to Dynamic Meteorology*, Academic Press, Inc., New York, 511pp.

Martin, J. E., and N. Marsili, 2002: Surface cyclolysis in the North Pacific Ocean. Part II: Piecewise potential vorticity diagnosis of a rapid cyclolysis event, *Mon. Wea. Rev.*, **130**, 1264–1281.

Montgomery, M. T., and B. F. Farrell, 1992: Polar low dynamics, *J. Atmos. Sci.*, **48**, 2484–2505.

Robinson, W. A., 1989: On the structure of potential vorticity in baroclinic instability, *Tellus A*, **41**(4), 275–284.

Rossby, C. G., 1940: Planetary flow patterns in the atmosphere, *Quart. J. Roy. Meteor. Soc.*, **66**(Suppl.), 68–87.

Thorpe, A. J., 1990: Frontogenesis at the boundarybetween air-masses of different potential vorticity, *Quart. J. Roy. Meteor. Soc.*, **116**, 561–572.

Wu, G.-X., Y.-P. Cai, and X.-J. Tang, 1995: Moist potential vorticity and slantwise vorticity development, *Acta Meteor. Sinica* (in Chinese), **53**(4), 387–405.

Zhang, D.-L., and H.-R. Cho, 1992: The development of negative moist potential vorticity in the stratiform region of a simulated squall line, *Mon. Wea. Rev.*, **120**(7), 1322–1341.

Zhao, S.-X., and S.-M. Fu, 2007: An analysis on the southwest vortex and its environment fields during heavy rainfall in eastern Sichuan Province and Chongqing in September 2004, *Chinese J. Atmos. Sci.* (in Chinese), **31**(6), 1059–1075.