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On the physical significance and use of a set of horizontal and vertical helicity budget equations

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ABSTRACT

For better understanding the variation of helicity and its governing mechanisms, based on the primary momentum equation under the local Cartesian coordinate, a set of horizontal and vertical helicity equations are derived in this study. On this basis, a storm-relative helicity budget equation is derived, the main factors that govern the variation of helicity are discussed, and the key mechanisms underlying the helicity variation are illustrated by using schematic images. Both scale analysis and real case diagnosis are used to compare the relative importance of different factors on the variation of helicity. For a meso-α system, it is found that: (i) horizontal helicity is much larger than vertical helicity, and they show significantly different variation mechanisms; (ii) for the vertical helicity, the vertical perturbation pressure gradient force, buoyancy, the divergence-related effect, and the conversion between vertical and horizontal helicity govern its variation (whereas, the conversion is negligible for the evolution of horizontal helicity); and (iii) baroclinity is crucial for the variation of horizontal helicity, but it is only of secondary importance for the vertical helicity variation.

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局地直角坐标系中水平与垂直螺旋度方程组的推导、物理意义及应用

摘要

本文基于大气运动方程组推导得到了水平与垂直螺旋度收支方程组，在此基础上，本文还率先推导了风暴相对螺旋度的收支方程组。本文利用物理图像揭示了不同影响因子（特别是水平与垂直螺旋度转化项）影响螺旋度变化的机理。利用经典的尺度分析与实际个例收支相结合的方法，本文着重讨论了中-α尺度天气系统水平与垂直螺旋度的演变，并确定了其各自的主要影响因子。这为相关的研究与预报提供了参考。

1. Introduction

In meteorology, the helicity is defined as $h = \boldsymbol{V} \cdot (\nabla \times \boldsymbol{V})$ Lilly (1986a, 1986b), where $\boldsymbol{V} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$ is the three-dimensional (3D) velocity vector, *u, v, w* are the zonal, meridional, and vertical velocity, respectively. $\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}$ is the 3D gradient operator, and ***i***, ***j***, ***k*** stand for the unit vector points to the east, north, and zenith, respectively. Helicity was first used in studies of turbulent flow (André and Lesieur 1977; Levich and Tsinober 1983a, 1983b), and it was proposed that helicity could resist dissipation (Moffat 1981). This hypothesis was verified by numerical simulations conducted by Pelz et al. (1985).

Lilly (1986a, 1986b) first used helicity to investigate disastrous weather. It was found that the effects of helicity could reduce energy loss and resist dissipation (Wu, Lilly, and Kerr 1992). It was also proposed that the effect of helicity on direct generation of the tornadic vortex might be important (Lilly 1990). After that, helicity was used in the forecasting of tornados (Woodall 1990), and was proven to be effective (Dupilka and Gerhard 2006; Clark et al. 2013). In addition to tornados, helicity was also found to be effective in forecasting other types of local severe storms (Tan and Wu 1994; Ding et al. 1996; Gao and Zhou 2006), including supercells (Weisman and Rotunno 2000), convective systems (Fei and Tan 2001), hurricanes (Han, Wu, and Fang 2006; Molinari and Vollaro 2008, 2010; Onderlinde and Nolan 2014, 2016) and heavy precipitation (Ran and Chu 2009; Wang et al. 2009).

From the above, as an effective forecasting factor for severe weather, helicity is widely used in meteorology. The state and variation of helicity provides a useful way to understand and forecast severe weather (Wu, Lilly, and Kerr 1992; Fei and Tan 2001). But through which mechanisms are the helicity produced/depleted? To answer this

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question, a budget equation of helicity is needed. Accordingly, many previous studies have derived helicity budget equations from different viewpoints (e.g., Lu and Gao 2003; Gao and Zhou 2006; Wang et al. 2009). However, to the best of our knowledge, the storm-relative helicity (SRH) budget equation has not yet been derived. Moreover, a physical picture of the mechanisms accounting for the helicity variation, particularly the conversion between the horizontal and vertical helicity, still remains vague. Therefore, the primary purpose of this study is first to derive a concise set of helicity equations, and then, based on the derivation, to provide an SRH budget equation and a physical picture of the key mechanisms governing the helicity variation.

2. Derivation of the helicity equations

The derivation is based on the primary momentum equation under the local Cartesian coordinate [i.e., Equation (1); Holton (1992)], where $\frac{\mathrm{d}}{\mathrm{d}t}=\left(\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla\right)$, t is the time, ρ is the density, p is the pressure, $f=2\Omega\sin\varphi$ is the Coriolis parameter (Ω is the rotational angular velocity of the earth, φ is the latitude), g is the acceleration due to gravity, and $\boldsymbol{F}$ is the frictional force:

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}=-\frac{1}{\rho}\nabla p-f\boldsymbol{k}\times\boldsymbol{V}-g\boldsymbol{k}+\boldsymbol{F}. \tag{1}$$

The 3D vorticity vector is expressed as

$$\begin{aligned}\boldsymbol{\zeta}&=\nabla\times\boldsymbol{V}\\&=\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\boldsymbol{i}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\boldsymbol{j}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\boldsymbol{k}\\&=\zeta_x\boldsymbol{i}+\zeta_y\boldsymbol{j}+\zeta_z\boldsymbol{k},\end{aligned} \tag{2}$$

where ζ_x, ζ_y, and ζ_z denote the vorticity component along the x, y, and z direction respectively. By taking $\nabla\times$ Equations (1) and (3) can be obtained:

$$\frac{\partial\boldsymbol{\zeta}}{\partial t}+\nabla\times(\boldsymbol{V}\cdot\nabla\boldsymbol{V}+f\boldsymbol{k}\times\boldsymbol{V})=-\nabla\alpha\times\nabla p+\nabla\times\boldsymbol{F}, \tag{3}$$

where $\alpha=1/\rho$ is the specific volume. By taking $\boldsymbol{\zeta}\cdot\mathrm{Equation}(1)+\boldsymbol{V}\cdot\mathrm{Equation}(3)$, the total helicity equation is obtained as Equation (4), where $\beta=\partial f/\partial y$ and $K=\boldsymbol{V}\cdot\boldsymbol{V}/2$ is the 3D kinetic energy (KE) of the unit mass:

$$\begin{aligned}\frac{\partial h}{\partial t}=&-\boldsymbol{V}\cdot\nabla h+\boldsymbol{V}\cdot(-\nabla\alpha\times\nabla p)+\boldsymbol{\zeta}\cdot\left(-\frac{1}{\rho}\nabla p\right)-g\zeta_z\\&+\left(fv\zeta_x-fu\zeta_y\right)-(\nabla\cdot\boldsymbol{V})(h+fw)\\&+\left[(\boldsymbol{\zeta}\cdot\nabla)K+f\frac{\partial K}{\partial z}\right]-\beta vw+(\boldsymbol{V}\cdot\nabla\times\boldsymbol{F}+\boldsymbol{\zeta}\cdot\boldsymbol{F}).\end{aligned} \tag{4}$$

From the definition of Wu, Lilly, and Kerr (1992), helicity can be decomposed into the horizontal and vertical components: $h=(\boldsymbol{V}_\mathrm{h}+w\boldsymbol{k})\cdot(\boldsymbol{\zeta}_\mathrm{h}+\zeta_z\boldsymbol{k})$, where $\boldsymbol{V}_\mathrm{h}=u\boldsymbol{i}+v\boldsymbol{j}$ and $\boldsymbol{\zeta}_\mathrm{h}=\zeta_x\boldsymbol{i}+\zeta_y\boldsymbol{j}$, since the total helicity is equal to the sum of the horizontal and vertical helicity ($h=h_\mathrm{h}+h_z$), and the horizontal and vertical helicity can be defined as $h_\mathrm{h}=\boldsymbol{V}_\mathrm{h}\cdot\boldsymbol{\zeta}_\mathrm{h}$ and $h_z=w\zeta_z$, respectively. The 3D KE of the unit mass can be decomposed as $K=(\boldsymbol{V}_\mathrm{h}+w\boldsymbol{k})\cdot(\boldsymbol{V}_\mathrm{h}+w\boldsymbol{k})/2=K_\mathrm{h}+K_z$, where $K_\mathrm{h}=\boldsymbol{V}_\mathrm{h}\cdot\boldsymbol{V}_\mathrm{h}/2$ and $K_z=\frac{1}{2}w^2$. Similarly, the solenoid term $\boldsymbol{\xi}$ in Equation (3) can be decomposed as:

$$\begin{aligned}\boldsymbol{\xi}=\nabla\alpha\times\nabla p&=\left(\frac{\partial\alpha}{\partial y}\frac{\partial p}{\partial z}-\frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial y}\right)\boldsymbol{i}\\&+\left(\frac{\partial\alpha}{\partial z}\frac{\partial p}{\partial x}-\frac{\partial\alpha}{\partial x}\frac{\partial p}{\partial z}\right)\boldsymbol{j}+\left(\frac{\partial\alpha}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial\alpha}{\partial y}\frac{\partial p}{\partial x}\right)\boldsymbol{k}\\&=\xi_x\boldsymbol{i}+\xi_y\boldsymbol{j}+\xi_z\boldsymbol{k}=\boldsymbol{\xi}_\mathrm{h}+\xi_z\boldsymbol{k}.\end{aligned} \tag{5}$$

Neglecting the friction effects and taking Equation (5) into Equation (4), the total helicity equation can then be written as shown in Equation (6):

$$\begin{aligned}\frac{\mathrm{d}h}{\mathrm{d}t}=&-\boldsymbol{V}\cdot\boldsymbol{\xi}+\boldsymbol{\zeta}\cdot\left(-\frac{1}{\rho}\nabla p\right)-g\zeta_z+f\boldsymbol{k}\cdot(\boldsymbol{\zeta}_\mathrm{h}\times\boldsymbol{V}_\mathrm{h})\\&-(h+fw)(\nabla\cdot\boldsymbol{V})+\left[(\boldsymbol{\zeta}\cdot\nabla)K+f\frac{\partial K}{\partial z}\right]-\beta vw\ .\end{aligned} \tag{6}$$

Following similar procedures, the vertical helicity equation is obtained as shown in Equation (7), where $\nabla_\mathrm{h}=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}$ is the horizontal gradient operator:

$$\begin{aligned}\frac{\mathrm{d}h_z}{\mathrm{d}t}=&-\zeta_z\left(\frac{1}{\rho}\frac{\partial p}{\partial z}+g\right)-w\xi_z-\beta vw\\&-w(\zeta_z+f)(\nabla_\mathrm{h}\cdot\boldsymbol{V}_\mathrm{h})+\boldsymbol{\zeta}_\mathrm{h}\cdot\nabla_\mathrm{h}K_z.\end{aligned} \tag{7}$$

Suppose $p(x,y,z,t)=\bar{p}(z)+p'(x,y,z,t)$, $\rho(x,y,z,t)=\bar{\rho}(z)+\rho'(x,y,z,t)$, where $\overline{()}$ represents the base state, which satisfies the hydrostatic relation, it can be derived that $-\frac{1}{\rho}\frac{\partial p}{\partial z}-g=-\frac{1}{\rho}\frac{\partial p'}{\partial z}-\frac{\rho'}{\rho}g$. This means the vertical acceleration can be decomposed into the vertical perturbation pressure gradient force (PPGF) and the buoyancy $\mathrm{B}=-\rho'g/\rho$. Taking the above relation into Equation (7), the vertical helicity budget equation can then be obtained as indicated in Equation (8):

$$\begin{aligned}\frac{\mathrm{d}h_z}{\mathrm{d}t}=&(\mathrm{PPGF}+\mathrm{B})\zeta_z-w\xi_z-\beta vw-w(\zeta_z+f)(\nabla_\mathbf{h}\cdot\boldsymbol{V}_\mathbf{h})\\&\qquad\mathrm{Z1}\qquad\mathrm{Z2}\qquad\mathrm{Z3}\qquad\mathrm{Z4}\\&+\boldsymbol{\zeta}_\mathbf{h}\cdot\nabla_\mathbf{h}K_z.\end{aligned} \tag{8}$$

CON

Subtracting Equation (7) from Equation (6), the horizontal helicity equation can be obtained as shown in Equation (9):

$$\frac{\mathrm{d}h_\mathrm{h}}{\mathrm{d}t}=\left[\zeta_x\left(-\frac{1}{\rho}\frac{\partial p}{\partial x}+fv\right)+\zeta_y\left(-\frac{1}{\rho}\frac{\partial p}{\partial y}-fu\right)\right]-\boldsymbol{V}_\mathrm{h}\cdot\boldsymbol{\xi}_\mathrm{h}+\left[(\boldsymbol{\zeta}+f\boldsymbol{k})\cdot\left(\nabla_\mathrm{h}K+\frac{\partial K_\mathrm{h}}{\partial z}\boldsymbol{k}\right)-h_\mathrm{h}\nabla\cdot\boldsymbol{V}-\boldsymbol{\zeta}_\mathrm{h}\cdot\nabla_\mathrm{h}K_z\right]. \tag{9}$$

The ageostrophic wind is defined as $\boldsymbol{V}_\mathrm{a}=u_\mathrm{a}\boldsymbol{i}+v_\mathrm{a}\boldsymbol{j}$, $u_\mathrm{a}=u-u_\mathrm{g}$, $v_\mathrm{a}=v-v_\mathrm{g}$, where u_g and v_g stand for the geostrophic wind. Therefore, Equation (9) can be rewritten as:

$$\frac{\mathrm{d}h_\mathrm{h}}{\mathrm{d}t}=f\boldsymbol{k}\cdot(\boldsymbol{\zeta}_\mathrm{h}\times\boldsymbol{V}_\mathrm{a})-\boldsymbol{V}_\mathrm{h}\cdot\boldsymbol{\xi}_\mathrm{h}+\left[(\boldsymbol{\zeta}+f\boldsymbol{k})\cdot\left(\nabla_h K+\frac{\partial K_h}{\partial z}\boldsymbol{k}\right)\right]$$

H1 H2 H3

$$-h_\mathrm{h}\nabla\cdot\boldsymbol{V}-\boldsymbol{\zeta}_\mathrm{h}\cdot\nabla_\mathrm{h}K_z. \tag{10}$$

H4 CON

Equation (10) is the horizontal helicity budget equation. The SRH is defined as $\mathrm{SRH}=\iiint_T(\boldsymbol{V}-\boldsymbol{C}_\mathrm{h})\cdot\nabla\times\boldsymbol{V}\mathrm{d}\tau$, where $\boldsymbol{C}_\mathrm{h}$ is the horizontal velocity vector of the system motion, and T is the volume of the system. The SRH density is defined as $\mathrm{SPHd}=(\boldsymbol{V}-\boldsymbol{C}_\mathrm{h})\cdot\nabla\times\boldsymbol{V}$.

Therefore, $\frac{\mathrm{dSPHd}}{\mathrm{d}t}=-\boldsymbol{C}_\mathrm{h}\cdot\frac{\mathrm{d}(\nabla\times\boldsymbol{V})}{\mathrm{d}t}-(\nabla\times\boldsymbol{V})\cdot\frac{\mathrm{d}\boldsymbol{C}_\mathrm{h}}{\mathrm{d}t}+\frac{\mathrm{d}h}{\mathrm{d}t}$.

Taking this relation and Equation (3) into Equation (6) and neglecting the friction, Equation (11) can be obtained as:

$$\begin{aligned}\frac{\mathrm{dSPHd}}{\mathrm{d}t}=&-\boldsymbol{C}_\mathrm{h}[(\boldsymbol{\zeta}+f\boldsymbol{k})\cdot\nabla]\boldsymbol{V}_\mathrm{h}-\boldsymbol{\zeta}\cdot\frac{\mathrm{d}\boldsymbol{C}_\mathrm{h}}{\mathrm{d}t}-(\boldsymbol{V}-\boldsymbol{C}_\mathrm{h})\cdot\boldsymbol{\xi}-(\nabla\cdot\boldsymbol{V})(h+fw)\\&+\left[(\boldsymbol{\zeta}\cdot\nabla)K+f\frac{\partial K}{\partial z}\right]-\beta vw\\&+\zeta_x\left(-\frac{1}{\rho}\frac{\partial p}{\partial x}+fv\right)+\zeta_y\left(-\frac{1}{\rho}\frac{\partial p}{\partial y}-fu\right)\\&+\zeta_z\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\right).\end{aligned} \tag{11}$$

Taking the ageostrophic wind, PPGF and B into Equation (11), the total helicity density equation is obtained as:

$$\begin{aligned}\frac{\mathrm{dSPHd}}{\mathrm{d}t}=&-\boldsymbol{C}_\mathrm{h}[(\boldsymbol{\zeta}+f\boldsymbol{k})\cdot\nabla]\boldsymbol{V}_\mathrm{h}-\boldsymbol{\zeta}\cdot\frac{\mathrm{d}\boldsymbol{C}_\mathrm{h}}{\mathrm{d}t}-(\boldsymbol{V}-\boldsymbol{C}_\mathrm{h})\cdot\boldsymbol{\xi}\\&-(\nabla\cdot\boldsymbol{V})(h+fw)+\left[(\boldsymbol{\zeta}\cdot\nabla)K+f\frac{\partial K}{\partial z}\right]\\&-\beta vw+f\boldsymbol{k}\cdot(\boldsymbol{\zeta}_\mathrm{h}\times\boldsymbol{V}_\mathrm{a})+\zeta_z(\mathrm{PPGF}+\mathrm{B}).\end{aligned} \tag{12}$$

Applying the 3D integral to Equation (12) within a system, the *SRH* budget equation can be derived as shown in Equation (13):

$$\begin{aligned}\frac{\partial\mathrm{SRH}}{\partial t}=&-\iiint_T\boldsymbol{V}\cdot\nabla\mathrm{SRH}\mathrm{d}\tau-\iiint_T\boldsymbol{C}_\mathrm{h}\cdot[(\boldsymbol{\zeta}+f\boldsymbol{k})\cdot\nabla]\boldsymbol{V}_\mathrm{h}\mathrm{d}\tau\\&-\iiint_T\boldsymbol{\zeta}\cdot\frac{\mathrm{d}\boldsymbol{C}_\mathrm{h}}{\mathrm{d}t}\mathrm{d}\tau-\iiint_T(\boldsymbol{V}-\boldsymbol{C}_\mathrm{h})\cdot\boldsymbol{\xi}\mathrm{d}\tau-\iiint_T(\nabla\cdot\boldsymbol{V})(h+fw)\mathrm{d}\tau\\&+\iiint_T\left[(\boldsymbol{\zeta}\cdot\nabla)K+f\frac{\partial K}{\partial z}\right]\mathrm{d}\tau-\iiint_T\beta vw\mathrm{d}\tau+\iiint_T f\boldsymbol{k}\cdot(\boldsymbol{\zeta}_\mathrm{h}\times\boldsymbol{V}_\mathrm{a})\mathrm{d}\tau\\&+\iiint_T\zeta_z(\mathrm{PPGF}+\mathrm{B})\mathrm{d}\tau.\end{aligned} \tag{13}$$

In real-case calculation, to determine an appropriate volume for the 3D integral is very important. The selection of the boundaries in the horizontal plane should (i) include the main body of the target system, and (ii) be insensitive to relatively small changes to its boundaries. The selection of the bottom and top boundaries should also satisfy the above two criteria.

3. Discussion on the helicity equations

3.1. Overview and physical significance of key terms

Since the vertical velocity *w* is generally much less than the horizontal velocity, and the horizontal scale of a system is generally much larger than its vertical scale, the vertical helicity is therefore generally much smaller than the horizontal helicity. However, it should be noted that for some very severe weather, such as tornadoes, h_z and h_h may be of comparable importance.

Although the vertical helicity is generally much smaller than the horizontal helicity, it is an effective indicator for the intensity of a system (Lu and Gao 2003). As shown in Equation (8), terms Z1–Z4 and CON (the conversion between horizontal and vertical helicity) govern the variation of h_z. Term Z1 represents the effects associated with the vertical acceleration, which can be further decomposed to the effects of PPGF and buoyancy. These two factors can enhance/weaken vertical helicity through accelerating/decelerating the vertical motions. Term Z2 is determined by the vertical motion and vertical solenoid (the vertical solenoid can modify the horizontal rotation that changes the vertical helicity), which is closely related to baroclinity. Term Z3 denotes the effect associated with advection of the planetary vorticity (it can affect the rotation in the horizontal plane), which is strong in tropical areas and weak in high-latitude regions. Term Z4 represents the effects associated with the stretching effect (Holton 1992), which usually acts as a key factor for vorticity variation. Term CON denotes the conversion between the horizontal and vertical helicity. To explain term CON, a simple configuration is used, as shown

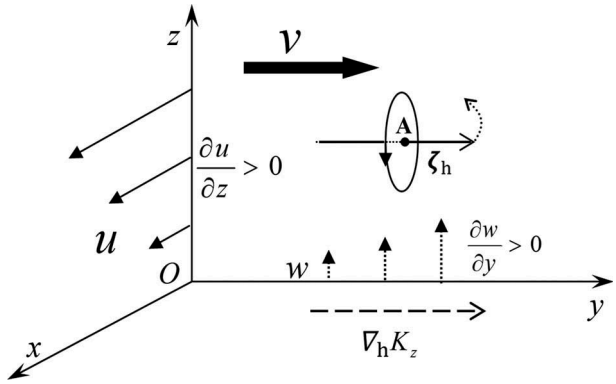

Figure 1. Schematic illustration of the conversion from horizontal helicity to vertical helicity, where the ellipse stands for the circulation associated with ζ_h, the thick solid arrow is meridional wind, short dashed arrows are vertical wind, the long dashed vector is the horizontal gradient of vertical KE, thin solid vectors stand for zonal wind, and the dashed curved vector represents the tilting direction of the horizontal vorticity vector.

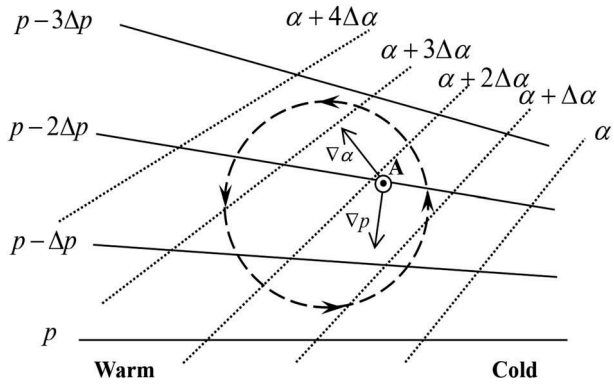

Figure 2. Schematic illustration of term H2, where the solid line is the isobar, the dotted line is isosteric, the dashed circle is the circulation caused by the solenoid, thin vectors are gradients of pressure as well as specific volume, and ⊙ stands for the solenoid which points out of the paper.

in Figure 1. Supposing the zonal wind $u = u(z)$ increases with height $\partial u/\partial z > 0$, and the meridional wind remains constant, only the horizontal vorticity $\boldsymbol{\zeta}_h$ will therefore exist, which is in the direction of $\boldsymbol{j}$, and the vertical helicity is equal to 0. Supposing there are perturbations of vertical motions (in reality, they are much smaller than the horizontal wind), which increase along the direction of $\boldsymbol{j}$ (thus $\boldsymbol{\zeta}_h \cdot \nabla_h K_z > 0$), the distribution of vertical velocity would induce the tilting of $\boldsymbol{\zeta}_h$ as the curved vector depicts; namely, horizontal helicity converts into vertical helicity. Generally, the horizontal helicity is much bigger than the vertical helicity, such that the horizontal helicity may be a big 'pool' of helicity for the vertical helicity. Under favorable conditions, the conversion from horizontal helicity to vertical helicity would benefit the enhancement/maintenance of a system. Therefore, the horizontal helicity could be a prognostic signal of severe weather (Lu and Gao 2003).

As Equation (10) shows, for the horizontal helicity, H1 is the ageostrophic effect term. This term vanishes for the geostrophic flow, or the horizontal vorticity vector and ageostrophic wind vector are perpendicular to each other. H2 is the horizontal solenoid effect term, which is shown in Figure 2 with a schematic model. Supposing there is inhomogeneous heating, which creates the distribution of the isosteric and isobaric surfaces, the gradients of pressure and specific volume at the location of an arbitrary point 'A' are shown with vectors. In this situation, the vector $\nabla\alpha \times \nabla p$ is pointing out of the paper, and the corresponding circulation is illustrated by the dashed circle. This circulation can modify the rotation in the vertical plane. If the wind has a component along the direction of $\nabla\alpha \times \nabla p$, the horizontal helicity would be modified by the circulation associated with the solenoid, whereas if the wind is perpendicular to the solenoid vector, the solenoid has no effect on horizontal helicity. Terms H3 and H4 indicate the absolute vorticity, 3D distribution of KE, and the variation of density can all influence the variation of horizontal helicity. The mechanisms associated with these terms are complicated and therefore need further explanation in the future.

3.2. Scale analysis

In this study, we use the meso-α scale, which is a typical scale for many commonly seen types of disastrous weather (e.g., southwest vortexes, typhoons, mei-yu fronts), to compare the relative importance of different terms in the helicity budget equations. According to the definition from Orlanski (1975), meso-α scale weather systems range from 200 km to 2000 km. For the meso-α scale, the estimates of the order of magnitude of each term in the vertical (Equation (8)) and horizontal (Equation (10)) helicity equations are as follows:

$$\frac{\mathrm{d}h_z}{\mathrm{d}t} \sim \mathrm{Z1} \sim \mathrm{Z4} \sim \mathrm{CON} \sim 10^{-9} - 10^{-10}\ \mathrm{m\ s^{-s}}$$

$$\mathrm{Z2} \sim \mathrm{Z3} \sim 10^{-11} - 10^{-12}\ \mathrm{m\ s^{-3}}$$

$$\frac{\mathrm{d}h_h}{\mathrm{d}t} \sim \mathrm{H1} \sim \mathrm{H2} \sim \mathrm{H3} \sim \mathrm{H4} \sim 10^{-6} - 10^{-7}\ \mathrm{m\ s^{-3}}$$

$$\mathrm{CON} \sim 10^{-9} - 10^{-10}\ \mathrm{m\ s^{-3}}$$

As shown above, regarding meso-α systems, for the vertical helicity equation, the main terms include Z1, Z4, and CON. This means the conversion between

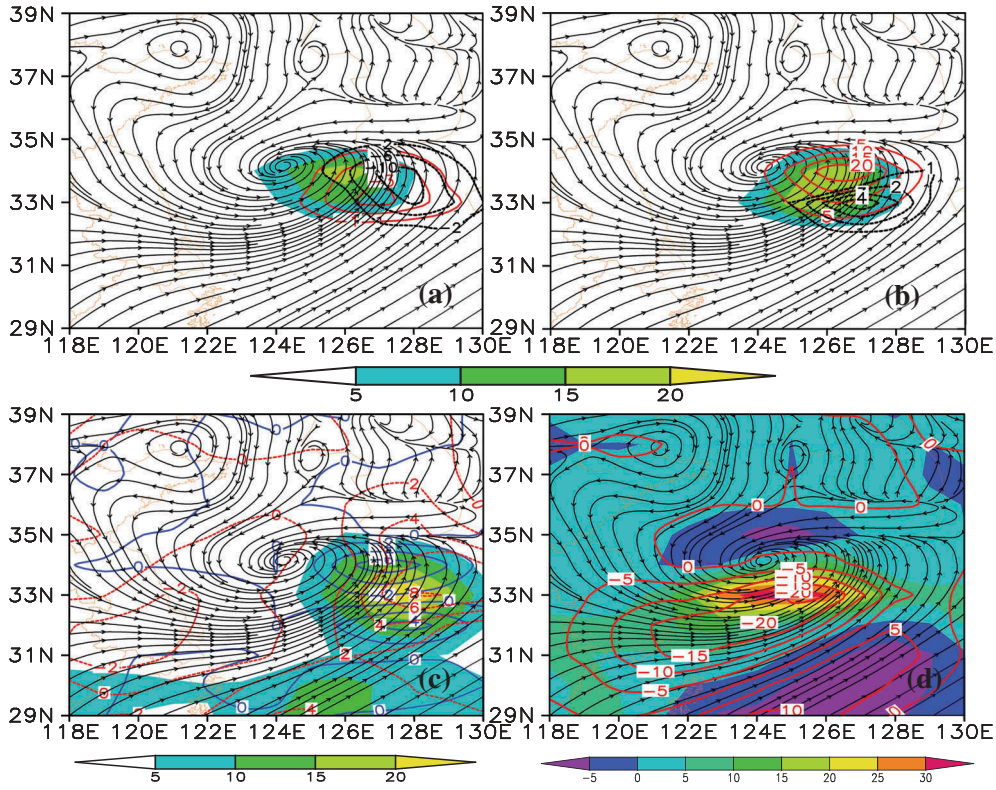

Figure 3. Terms of the vertical and horizontal helicity budget equations at 900 hPa during the mature stage (1800 UTC 3 July 2007) of a meso-α-scale vortex, where the solid line is the stream line. The shading in (a) is vertical helicity (units: 10^{-6} m s^{-2}), the black dashed lines are term Z2 (units: 10^{-12} m s^{-3}), and the red solid lines are term Z3 (units: 10^{-11} m s^{-3}). The shading in (b) is term Z1 (units: 10^{-10} m s^{-2}), the black dashed lines are term CON (units: 10^{-10} m s^{-3}), and the red solid lines are term Z4 (units: 10^{-10} m s^{-3}). The shading in (c) is horizontal helicity (units: 10^{-2} m s^{-2}), the red dashed lines are term H1 (units: 10^{-6} m s^{-3}), and the blue solid lines are term H3 (units: 10^{-6} m s^{-3}). The shading in (d) is term H4 (units: 10^{-6} m s^{-3}), and the red solid lines are term H2 (units: 10^{-6} m s^{-3}).

horizontal and vertical helicity is important to the variation of vertical helicity. In contrast, term Z2 is much smaller than the main terms, implying that the effect related to baroclinity is only of secondary importance for the variation of vertical helicity. Since β is very small, term Z3 is also much smaller than the main terms, which can be ignored. However, it should be noted that the relative importance of term Z2 (which is closely related to baroclinity) would increase as the horizontal scale increases.

For the horizontal helicity of meso-α systems, the main terms are H1, H2, H3, and H4; whereas, term CON is approximately three orders of magnitude smaller than the main terms. This implies that the conversion between horizontal and vertical helicity is negligible in the variation of horizontal helicity. However, it should be noted that the relative importance of term CON would increase as the horizontal scale decrease. For meso-γ scale systems (which range from 2 to 20 km), term CON is of the same order of magnitude as the other main terms. Different from the situation for vertical helicity, the baroclinity-related effect (i.e., term H2) is of the same order as the other main terms, which indicates that baroclinity is important to the variation of horizontal helicity.

3.3. Real case budget

In order to verify the results from the scale analysis, based on the final analysis data from the NCEP with a resolution of 1° × 1°, we calculated the vertical and horizontal helicity budgets of a meso-α vortex. This vortex occurred over the Yangtze River valley during 0600 UTC 3 July to 0600 UTC 4 July 2007, and induced heavy rainfall during its mature stage. As Figure 3(a) shows, the vertical helicity could reflect the convective zone associated with the vortex well, as in this event intense convective activities mainly occurred in the eastern section of the vortex (not shown). Terms Z1, Z4, and CON were the main terms producing vertical helicity (Figure 3(b)), whereas terms Z2 and Z3 were generally smaller and mainly acted to reduce the vertical helicity (Figure 3(a)). From Figure 3(c), it can be

seen that positive horizontal helicity was also closely associated with the convective activities. Compared to the vertical helicity, the horizontal helicity was much larger in both order of magnitude and horizontal range. Within the area characterized by positive horizontal helicity, term H1 mainly enhanced the horizontal helicity, while term H3 mainly acted in an opposite way. Terms H2 and H4 were more intense than terms H1 and H3 (Figure 3(d)); however, because these terms canceled each other out intensely, their summed effect was smaller than their respective effect. Overall, it can be concluded that the main features and the orders of magnitude of the budget terms of both the vertical and horizontal helicity equations are consistent with the results derived from the scale analysis.

4. Conclusions

In this study, a concise set of vertical and horizontal helicity budget equations have been derived on the basis of the basic momentum equations under the local Cartesian coordinate. On this basis, an SRH budget equation is derived, and the key mechanisms governing the helicity variation are discussed. These equations can be used in severe weather diagnoses, such as typhoons, mesoscale convective vortices, tornados, etc., as there is no hydrostatic balance hypothesis. The concise set of vertical and horizontal helicity budget equations and the SRH budget equation derived in this study can be used in the diagnosis of severe weather. As the vertical and horizontal helicity budget equations indicate, the horizontal and vertical helicity show very different variation mechanisms, with a conversion term (i.e., CON) linking them together. A physical picture of the conversion is shown in Figure 1. Regarding meso-α scale weather systems, both the scale analysis and real-case calculation show terms Z1, Z4, and CON in Equation (8) act as the main factors for the vertical helicity variation, whereas Z2 and Z3 are negligible. In contrast, for the horizontal helicity, terms H1, H2, H3, and H4 in Equation (10) are the main factors, whereas CON is much smaller. Because the horizontal helicity is much larger than the vertical helicity, the former can serve as a source of helicity for the latter under favorable conditions. At least for the mesoscale vortex in this study, the conversion from horizontal helicity to vertical helicity (i.e., term CON) is vital for the evolution of a meso-α scale vortex with heavy rainfall.

Disclosure statement

No potential conflict of interest was reported by the authors.

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